Unschooling Math

How homeschoolers, ages 4 - 17 learned math in unconventional ways.

Selected from *Growing Without Schooling* magazine by Susannah Sheffer

REVELS IN NUMBERS

From Julie Loyd (CA):

Charlie (4) has always liked numbers. I'm good at math but distrust it. Charlie revels in numbers. When he was tiny, he used to run around the house shouting "Wah!" Much later, when he became comprehensible, we realized that he was counting, but could only count up to one.

As soon as he could put together two-word sentences he'd count things, this time up to two. "One light!" "Two people!" "Two baby-goat!" "One mommy-goat!" For a couple of years, he seemed to be content with this. Then recently, when his 4th birthday was coming up, he decided that 4-year-olds ought to know how to count. He began asking me to identify every number he saw. We'd be driving in the car. and he'd ask, "Mom, what's a one and a four and a six and a three and a zero and a one?" "146,301," I'd say. (I'm pretty good at this. now.) He asked me to count up to 100, over and over again. Charlie doesn't like to sing or chant, so he'd never join in or attempt to fill in if I suddenly petered out to make a left turn or something. But after a few weeks, he'd yell at me from where he was playing with his trains, "Mom, what comes after 79?" and I'd know he'd been counting to himself. He's counted for me a few times recently, and although he gets mixed up in the teens, he can get to 100 quickly and accurately.

Occasionally he demonstrates to me that he adds small amounts as a matter of course. He'll say, "I'm gonna draw eight turbines on this plane, four on one wing and four on the other. No, wait. I'll put two on each wing and two on the tail and two on the nose. Isn't that ridiculous?" I hope he never finds out that this is supposed to be difficult. Now he is working on really big numbers. "Mom, you know how many I love you?" "No, how?" "A googol! A googoldy thousandy sixty five eleven two thousand." I can't wait to see where this leads.

CRAZY QUESTIONS

From Kathleen Plunkett-Black (WI):

One of the things we enjoy doing is figuring out the answers to crazy and "irrelevant" questions that come up, often at mealtimes, so the kids can see us doing math just for the fun of it. "How many peas would it take to fill up this whole table and the space under the table?" Christopher (5) asked one day last summer as we were rationing out the small first harvest of fresh peas. "Well, how big is one pea?" we asked. and with that and the size of the table to work with, we came up with about 56,000 peas.

"How big would a tape measure long enough to reach from here to Pepin (10 miles away) be if it was rolled up?" Lorin (8) asked one morning, and got to see us both get interested and challenged by the problem, try various ways of looking for the answer, get frustrated when they didn't work, and finally write to my mother, a retired math teacher, for

the right formula to solve it. Our answers are always approximate, since we round numbers off a lot as we go, and try to focus on the problem-solving process rather than the arithmetic. Another advantage is that Bryce [their father] and I usually use different methods to get the solutions, so the children get to see that there isn't only one way to do it.

TEACHING HIMSELF ARITHMETIC

From Maggie Sadoway (MA):

When my son Solon was 3 or 4 and I was first reading letters in GWS from parents whose children had taught themselves arithmetic, I was highly skeptical. "These parents are exaggerating," I would think to myself. "Certainly some children teach themselves to read, but what child would reinvent multiplication tables or figure out borrowing and carrying?"

But the process was already well underway in my own house. Solon, now nearly 9, has continued to be as fascinated with numbers as he was when I wrote about him in GWS #48. He has definitely taught himself arithmetic with very little help from anyone.

From time to time I enjoy rereading Nancy Wallace's thought-provoking article,"Why Study Math" (GWS #54). While music mystifies me as math does her, I am very good at complicated basic arithmetic and rate trigonometry as my favorite high school subject. Nancy's son Ishmael's main passion is music; my son is passionate about numbers. Although it is not obvious that he "practices" arithmetic several hours a day as Nancy's children do music, I see many signs that he constantly views the world through a filter of numerical relationships. No one has taught him to do that; as he has gradually shown me, we live in such a number-rich world that any child whose interest is sparked can find interesting problems to solve all day long, just as Solon has done.

Here are some of my favorite examples:

(Age 5): Solon is playing a game with some soup cans at the natural foods store we own. I watch closely without him noticing me. "Six take away one" (he slides one can to the other side of the shelf) "leaves five. Five take away one (slide) leaves four" At "one take away one" there is a long pause. I struggle not to say, "But you know that! What's 1-1?" It is one of my proudest moments as a parent that I say nothing. A moment later I get my reward: a glimpse of how exciting numbers are to him. "One take away one" - dramatic pause followed by a rich, triumphant shout - "leaves ONE MILLION, the curtain rises and the show begins again!"

(Age 6, traveling in Greece): "Tickets are 10 drachmas each and there are 4 of us so that's 40 drs. And if each of the 3 of you makes 2 round trips with me on this little ferry (a 3-minute ride between island and mainland) I'll get to go 12 times." "I made up a game where only a quarter of the people speak English, so a quarter and a half speak Greek."

(Age 6, unsolicited comments while listening to stories): "Since it

takes 2 hours to fill one, they could fill 5 in 10 hours." "If oranges are 5 for 10 pence, they cost 2 pence each." "You say it's five minutes to eleven? I say 10:55."

(Age 7): "This costs $2.75 at half price so I just figured out it used to cost $5.50." "Mom, if 4 people in the class you teach each pay you $50 and you get to keep three quarters of the money, you'd have $150. That's easy for me to do in my head." "Can I have 6 potato pancakes? There are only 9 for the three of us? (He continues without the slightest pause.) "OK, so you and Jack can each have 1 1/2."

(Age 8): "Mom, did you know that when there are 4 people, like you, me, and the two kittens, that you can make six arrangements? There's you and me, you and Fudge, you and Caramel, me and Fudge, me and Caramel, and Fudge and Caramel." After the light was out one night, I heard Solon ask himself, "How many tens would there be in a million? I know there are 1000 thousands, so there'd be 10,000 hundreds, 100,000 tens and a million ones. That means there'd be half a million twos, a third of a million threes, a quarter of a million fours, a fifth of a million fives and a sixth of a million sixes."

The only time Solon writes numbers down is for phone numbers and keeping score in games. He rarely uses a calculator and never does his figuring on paper - I'm not sure he knows how. He has never used textbooks, flashcards, or worksheets, and we hardly ever suggest problems for him to figure out. For a while he was an avid fan of "3-2-1 Contact" and "Square One," two TV programs with lots of arithmetic. Because of the way he works, I have very little useful information about how he has actually learned all this arithmetic.

But come to think of it, I have little useful information about how he's learned most of the million and one things he knows. How did he learn to walk? How does he remember the names of at least fifty transformers? Why did he know how to tie his shoes one Monday morning when he'd been totally frustrated in his attempts only the day before?

I think we do young people a great injustice by constantly underestimating how enormously intelligent and capable they are, how eager they are to learn all they possibly can, how hard they're willing to work to know just one more thing, how easy it usually is for them to learn almost anything they're genuinely interested in. We notice that adults have much more information about the world, and somehow think this means children aren't as smart. Confusing knowledge and intelligence in this way leads us into crossing that ever so fine line between rightfully admiring competency in anyone of any age. and being sexist ("She's a good mechanic for a woman") or adultist ("For a 6 year old you sure are good at that.") I fully understand why Solon loathes our customers who tell him, usually in a sappy tone of voice, "Oooh, you must be really smart to run a cash register at your age."

I've also noticed that we tend to be most surprised about what children have been able to learn when they master something that most people think of as being difficult (whether it really is or not) or where we ourselves have been made to feel stupid or bad about ourselves. Thus we calmly accept that children know how to use telephones or typewriters

but are astonished when they figure out how to add two-digit numbers on their own or work a computer.

I am reluctant to ask Solon many questions about his methods of doing arithmetic for fear of tampering with his elegant system of teaching himself. I do know that he's had lots of experience with money at our store, with his personal purchases and bank account, and through his twice-a-year used toy sales where he regularly takes in over $100. (He keeps track of the original price he pays for toys, adds on the sales tax he paid, subtracts 40% with a calculator, then rounds off the answers to find the amount he wants to sell them for.) I also know that he sees us use numbers a lot and that he's never been given reason to think arithmetic is hard or beyond him. Once in a while, because he knows I'm interested, he'll share with me how he's gotten a certain answer. A few examples: "600 drachmas must be $4 because 300 drs. is $2." "9 + 7. Nine is 2 more than 7. So put 1 on the 7, that's 8 + 8 = 16." "Five 15's are 75? Oh, yes, of course, because 20 fives are 100 and five 5's less would be 75."

Solon's days are full of a variety of things he enjoys: friends, art, computers, his bike, swimming, many toys and construction sets, being read to, writing stories and poems, studying maps and turnpike toll cards, etc. As far as I know, he never categorizes his activities by thinking of them as geography or arithmetic or astronomy. I don't think he notices that he's good at arithmetic any more than he thinks about how competent he is at breathing or speaking English. If he were asked to name his chief interests, it's unlikely that he would even mention numbers. They are simply a very interesting but taken-for-granted piece of his life, like the sky above and the ground below.

In one way I was right to scoff at the idea of children teaching themselves arithmetic, but only because I somehow assumed they would still learn it in the way it's presented in school, with lots of strange rules and boring drills. Children who teach themselves to read almost certainly don't do it by saying things like, "Let's see, the silent e on the end of this word makes the vowel say its name," nor do children teach themselves to multiply by reinventing the multiplication tables before they start. Solon can easily add numbers such as 47 + 58, but he doesn't do it the "proper" way. (Even the proper way depends on what year you were born or what culture you live in. Solon has discovered it is much easier to add 29 + 48 if you think of it as 30 + 50 minus the 3 you just added on. In Greece I have seen children taught that method in school but as far as I know it is not common in this country.) Solon solves problems by using a system he's figured out for himself, one that truly makes sense to him My guess is that in the process he's made his knowledge about numbers his own, a permanent part of his life, in a way that would be the envy of any school program.

Perhaps most important of all, he not only thoroughly enjoys himself as he figures all this out, but gets to feel proud and pleased about himself at the same time - the very best foundation there is for ever more learning.

Naughty and Disgusting Math

Phoebe Wells (MA) writes:

Our family enjoyed both Carla Stein's (GWS #110) and Susan Axe-Bronk's (GWS #112) descriptions of their families' math adventures. We too have found that the real math of our lives and the things we love is the best way to enjoy numbers. For us, math is either a game or another way of understanding something or both.

We first noticed Eoin's interest in numbers when he was 4 and immersed in the details of *Custer's Last Stand.* What was the average number of braves per Native American chief? And how did that compare to the number of soldiers in Custer's troops? Even after Eoin had his fill of this subject, he was still hungry for more math challenges (he never liked the term "math problem," because it doesn't sound fun at all). However, at the time, I was still in that bleary postpartum stage after the birth of my second child, Daire, as well as coping with a family crisis or two, so I was not at my creative best. I tried to think up interesting scenarios for 5-year-old Eoin about dividing up oranges and sharing toys, but I'll be the first to admit they were pretty dull. Still Eoin pleaded for more, so finally in a desperate moment I thought up a gem based on recent experience: if Eoin has stomach flu and throws up 6 times in one night but only makes it to the bathroom 2/3 of the time, how many times did Mama need to change the sheets? Eoin was delighted, and "disgusting math" was born.

Life with an infant offered plenty of real-life situations which we examined in this mathematical fashion, and as Daire grew, his increasing abilities presented more food for thought. This became fondly known as "naughty baby math": if we give Daire 10 banana slices for lunch and he flings 2/5 of them on the floor and smooshes another 1/5 in his hair, how many did he actually eat?

Eoin's mathematical abilities flourished in this environment, and we moved on to improper fractions, decimals, and percents. For example, if Eoin has just helped his exhausted mother by folding 40 diapers into a nice, neat pile and Daire toddles over and scatters 50% onto the floor and throws another 10% behind the couch, how many diapers must Eoin retrieve and refold? Time and motion studies were popular as well: if Eoin has just built a huge block tower in the living room and Daire spots it from the kitchen and begins crawling toward it at a rate of 10 inches per second, how long does Eoin have to kiss his creation goodbye?

To be fair, this wasn't the only math Eoin chose to do. When the World Cup played here, Eoin discovered the tremendous amount of numbers in sports. What fraction of the total number of teams playing in the Cup would make it to the finals? Based on its record, what are a particular team's chances of winning, compared to its opponent's? At 9, Eoin has now branched out to other sports and loves all the statistics involved.

We've also been delighted to find a variety of books at the library that showed the kids more ways to play around with numbers and mathe-

matical ideas. Some picture books we enjoyed are *Remainder of One,* by Elinor Pinczes (about division), *Sea Squares,* by Joy Hulme (about square numbers), *The Rajah's Rice,* by David Barry (about doubling numbers), and *The Tangram's Magician,* by Lisa Campbell Ernst (about tangram puzzles). Eoin also liked The Young Math Book series, especially *Estimation,* by Charles Linn and *Base Five,* by David Adler. Daire's favorite, when he was old enough, was *Angles Are Easy As Pie,* by Robert Froman. Recently, Eoin has been inspired by *Math Equals: Biographies of Women Mathematicians* and *Related Activities,* by Teri Perl. Specifically, he was interested in the number patterns that inspired Ada Byron Lovelace, who became the world's first computer programmer in 1843.

At 9 and 4, now, the boys keep discovering more ways to enjoy numbers (it's geography and Cuisenaire rods at the moment), but still they request and rejoice in new "disgusting/naughty math" that reflects our present lives: if Eoin and Daire persisted in playing a wild game of soccer in the living room even though Mama told them 20 times in one afternoon to tone it down, and someone got hurt 1/4 of those times, how many times did Mama snap, "I told you so"?

Finding Mentors Who Love Math

Joanna Hoyt (ME) writes:

From a very early age I was interested in everything. I wanted to know why this and why that, wanted to understand and to know about all the different answers to my questions. I had many questions about math – about numbers, about patterns, about shapes and their names. I remember proudly learning "trapezoid" and "pentagon." I learned addition and subtraction on my own before I would have been considered "school age," with the help of Cuisenaire rods and sometimes explanations from my mother. With the rods, I could see easily not only that 2 plus 2 was 4 but why it was 4. I also learned the basic principles of multiplication with the rods – it was clear that three red rods were as long as one black rod, or two purples made a brown.

The problem came when I was 8 and my mother remembered that she had been 8 years old when she, in school, learned the times tables. I was ahead in other subjects, and had learned some math early, but I did not want to learn times tables. Poems or things that interested me I could memorize easily. I was not interested in learning lists of equations. There was only one right answer, and they were rather dull anyway. I did not learn my times tables well. I also decided that I hated math. I was still interested in shapes and patterns, and I delighted in logic puzzles, but I did not think of these as Math.

Fortunately, before this could go on for long, my mother, already worried about my stubborn refusal to learn multiplication, went to a conference and came back rather relieved. I'm not sure what she had been told, but we dropped times tables and she taught me how to break down a number into its prime factors. That looked interesting to me. There was still only one right answer, but it took some thought and it did not

have to be memorized. It was like a puzzle.

Having mastered prime factors, I proceeded on to long division. I had a book, a large gray arithmetic book, that my mother wanted me to finish. Long division again was boring – the multiplying, subtracting, dividing over and over was tedious, and I was not interested. I kept doing problems over and over because I was getting them wrong. After what felt like ages of this, I was convinced that my brain was incapable of doing math.

My mother was alarmed and concerned over this, and one day she sat down with me to talk about it. I was mad – I said I couldn't do it and I hated it, trying not to cry. She saw that I was almost crying. I don't remember exactly what she said, but soon she had me writing out what I was saying to myself about math. This idea came from Transactional Analysis, which my mother had introduced me to quite early on. TA uses a three-part model of each person: a Parent that contains shoulds, oughts, and how-tos, a Child that contains emotions, imagination, fantasies and wishes, and an objective Adult that contains information about the world outside and inside yourself. My dialogue about math was between an angry Parent and a whimpering Child. It began with the angry Parent accusation: "You're supposed to be smart, and you can't do basic math! What are you, stupid? Why don't you just shape up and do it right!" And the Child wailed back, "I can't do it – it's too hard – it's boring anyway – it isn't my fault!" Several rounds of this came out on the paper. I was shocked – I hadn't stopped to listen to what I was saying to myself. I added a last piece onto the dialogue, spoken by the Adult, reassuring me that I could do math; it was just harder for me than some other things were, and it wasn't my favorite subject. That was OK. I kept the paper in the back of my arithmetic book and looked at it when I was discouraged.

I finally finished long division and worked through decimals, which weren't very interesting as the textbook presented them, but were OK. In the back of the book was a little bit of very basic algebra. I was interested once again. Variables! Unknowns! It sounded interesting to me, and at first it was. But as the weeks wore on it began to seem less and less so. It seemed repetitive and dull again, but I kept telling myself, truthfully, "It's better than arithmetic." I scored poorly on the tests, though, because there were too many problems and I got careless with them. Math was OK now, but still not really fun. Algebra dragged on and on.

The January after I started algebra, my family went down to a conference in New York. There were various workshops on writing, science, dance, art, and math. My mother encouraged me to try the math workshop, and I agreed. Algebra had given me a little more interest in math, and I thought the workshop might be worth a try. I was nervous, though; the conference was meant to be for gifted children and their parents, and I was afraid that I would arrive there and seem really dumb.

The math in the workshop was like nothing I had been exposed to before. The workshop leader, a graduate student who had switched his undergraduate major from English to math, talked about math that had more than one answer. He explained that mathematicians still disagreed

on whether the set of whole numbers was twice as large as the set of odd numbers (since only every other whole number is odd), or the same size (since both sets are infinite). All of us took sides and argued about it. I could understand it, and it was enjoyable – even fascinating! He introduced me to Euhler's law, which says that for any 3-dimensional shape made with straight lines, the number of its vertices (places where lines meet) added to the number of its faces (sides) was always 2 more than the number of lines in the shape. I constructed a shape that I said didn't fit the law. (It did fit, but only if you accepted the rule that faces could not have holes in them.) I thought this fascinating, and spent the lunch hour with him counting up the lines and faces on a stemmed dessert dish.

When we went back home, I kept up a correspondence with this workshop leader, and we are still in touch. I have since read some Euclid, and am playing with tilings – shapes with which one could solidly cover a flat plane without wasting space between edges. (They don't have to come out square at the edges – triangles and hexagons, for example, are acceptable, but circles aren't, since they cannot share edges with one another. And then there are patterns of many shapes.)

Now I am about to start calculus with the graduate student's help. Higher math is FUN. And I've even learned some interesting paradoxes in arithmetic! I am beginning to see now how math is used in art (not just the angles and shapes of the actual picture, but lines of motion and compositional areas), in nature (leaves, dogs, sea urchins all have symmetry of one kind or another; leaves alternate or spiral up around branches), and in music (two, three, or four beats in a measure; actual notes are various fractions of the beats), and some kinds of poetry (meter and rhythm and length).

My brain is capable of doing math. And it enjoys math, too.

Unschooling Math With Teenagers

Aaron Falbel of Massachusetts writes:

A few months ago, I was contacted by a 16-year-old homeschooler named Anna who said she wanted to learn about mathematics in an unschooled manner. I had been saying for some time how unfortunate I think it is that when it comes to math, especially when dealing with teenagers, many homeschoolers of the unschooling variety stick very close to the conventional textbooks, which are anything but unschooled.

So when Anna called me, saying that she was particularly interested in mathematical thinking and in relating mathematics to her other interests, I couldn't say no. Here was my chance to put my money where my mouth was, so to speak, and demonstrate what I meant by real math as opposed to school math, as I had discussed in GWS #63 ("The Mathematics of the Ordinary").

When Anna arrived, she brought a friend named Ariel, who was also new at homeschooling and similarly interested in doing something different with mathematics. So we were a threesome.We agreed to meet for

weekly sessions lasting about two hours.

Early on in our meetings, I gave Anna and Ariel a written statement titled "Some Preliminaries," in which I described my take on mathematics. I felt it was important that we were all clear about what we were getting into and that Anna and Ariel had an opportunity to voice any objections to what I had proposed. After all, I could only agree to act as a teacher or helper if what I had to offer was indeed what they wanted. Here's some of what I said:

1. Mathematics is more than computation or arithmetic. It is by no means limited to "working with numbers."

2. In my mind, the only reason (or the only good reason) for studying mathematics is that you think it is intriguing, interesting, and fun. If you don't find it to be any of these, then my advice is leave it alone. In this respect, mathematics is no different from music. No more than with music do I believe that you should study math because "it'll be good for you." If you approach math in this for-my-own-good way, then chances are you will learn to hate it.

3. I believe mathematics is one way of seeing the world – one way among many, I hasten to add. It is more about thinking about patterns than thinking about numbers.

4. School math is very different from real math. School math is mostly about computation (arithmetic) and symbol manipulation techniques. By themselves, these things can be awfully boring. Real mathematicians do not sit around all day doing school math. School has concentrated on this one tiny part of mathematics because: a) it can be graded easily, and b) most school teachers are not mathematicians and have little or no idea what mathematics is really about or what real mathematicians do.

5. So I propose that we "skip over" the boring stuff and go straight for the fun stuff. (I consider fun stuff to be: logic puzzles, paradoxes, topological puzzles, geometry, probability, recursion puzzles, etc.) I put the words "skip over" in quotes because in reality we may see some school math creep back in as we try to deal with the fun stuff, but only if it helps us think about the fun stuff more clearly and carefully.

6. It is vitally important, if I am to be of any help to you, that you tell me when you don't understand something. There is no such thing as a stupid question! (Well, maybe quiz questions are stupid questions because they are rude, impolite, and invasive.) Likewise, you must tell me if you don't think something is fun. People have different tastes in music, and the same is true for mathematics. Just because I think something is fun doesn't necessarily mean that you will think so too. You must let me know what appeals to you and what doesn't. Remember, you are in the driver's seat here. I am just coming along for the ride, though I am happy to point out some interesting sights along the way.

7. It is a total myth that women are not as good as men at math. I know plenty of women who are far better mathematicians than I am. It is true that many women are socialized to think that they are not good at math, but I believe this has started to change. It may be true that many women think about mathematics in a somewhat different way than most

men do, but this is not always the case. And "different" does not imply "worse" or "better." (Historically, women have been excluded from mathematics for reasons having to do more with sexism than with any real lack of ability.)

8. I will be happy to bring in examples of puzzles or books that I think are intriguing, interesting, and fun, but I strongly encourage you to do the same. Browse through stuff at the library or at a bookstore, or at places that sell puzzles and games.

9. How often we meet, and for how long, is totally up to you (providing, of course, it fits into my schedule).

Their reaction to this document was, "Cool!"

For our first session, I brought in some books by Raymond Smullyan (*The Lady Or The Tiger* and *What is the Name of This Book?*) consisting of all sorts of logic puzzles. These books are about as far away from math textbooks as you can get. They are chock full of amusing puzzles – pure mathematical candy. The puzzles are not meant to illustrate important mathematical principles; they are simply fun. But they are incredibly rich, mathematically speaking. The puzzles escalate in difficulty as the book progresses, and solving them requires careful, rigorous, systematic thinking – in other words, mathematical thinking. Cultivating this style of thinking was at the core of everything Ariel, Anna, and I did in our sessions. Mathematical concepts and techniques were addressed, but as a side dish to the main entree of learning to think mathematically, as mathematicians do. (Incidentally, I brought in some biographical material on Raymond Smullyan. As it turns out, he was largely a self-taught mathematician who dropped out of school several times.)

In addition to looking at many of the fine books of mathematical puzzles and games by Martin Gardner, we also made use of the excellent books by Marilyn Burns (*The I Hate Mathematics! Book* and *Math For Smarty Pants*). For example, we played and analyzed the game of Nim, discussed in the Burns book. To succeed at this game, you have to learn how to reason backwards – an important part of mathematical thinking. Aside from these being among the best books I know that relate what real math is about, I felt it was important that Anna and Ariel see math books written by a woman and that math is not exclusively a male domain.

At another session, I introduced them to some topological puzzles involving rope and string. (Topology, unlike geometry, deals with the overall properties of shapes as opposed to their exact measurements. Twisting, stretching, and distorting – but not cutting! – are all allowed.) One puzzle involved tying the two of them up! There was much laughter that day. The topological puzzles were by no means easy; they had in the past stumped more than a few adults. Some puzzles they took home with them to think about for a week. But with a few hints here and there, Anna and Ariel were able to work through most of them.

A small Tower of Hanoi puzzle. The goal is to transfer all the disks to another pole, moving one disk at a time. A disk may never sit on another disk that is smaller than it is. These puzzles can be found in many math game catalogs; you can also make your own using jar lids or anything else. Illustration from *Metamagical Themas*, by Douglas R. Hofstadter

One of the Smullyan books had a few

algebra problems in it, and Ariel and Anna expressed a desire to work on problems of that sort. I wasn't quite sure if they were genuinely interested in such problems or whether they wished they could master that part of school math. We spent a couple of sessions working on algebra problems in one and two variables (largely ones I made up) solving them both algebraically and confirming these solutions using plain old "common sense." I'm not sure we did enough of these problems, though. I got the feeling that the algebraic method still seemed rather mystical to the two of them, somehow magically producing the right answer. It takes more time than we had for this type of understanding to sink in. (It may yet happen one day.)

Other sessions involved solving the Tower of Hanoi puzzle (a recursive puzzle). Working cooperatively, Anna and Ariel solved an 8-level tower, which took 255 moves. We spent one session talking about how computers make use of binary numbers and solving a few "tricky" physics problems. We also looked briefly at a book by the Japanese author and illustrator Mitsumasa Anno, which turned the story of "The Three Little Pigs" into an amazing *tour de force* of combinatorics and permutations.

At this point Anna became busy with a theatrical production and we stopped meeting. But we had planned to look at the connections between mathematics and art (one of Anna's main interests) which would lead us into the Golden Ratio, Fibonacci series, the discovery of π, trigonometry, and so on.

An outsider looking at us wearing school-colored glasses might deny we were doing math at all. After all, I had not given the two young women a single worksheet or problem set. We did solve some real mathematical problems, but most of this took place in the context of solving puzzles or playing games. But I believe that what we did came a good deal closer to what real mathematicians do than what passes for mathematics in the classroom (and, alas, in many homeschools).

Though I can't prove it, it is my belief that these two young women learned something important about mathematical thinking – not by hearing me lecture about it but by actually engaging in it. Though we only met for about six or seven sessions, I believe Anna and Ariel walked away with a better understanding of what it might be like to be a mathematician – to wrestle with an intriguing problem for a while and feel the joy of working toward a solution. I hope they came to see mathematics less as the sort of necessary baggage school people say one ought to carry around and more as a way of looking at and exploring the amazing variety of patterns in the world around them, as an experience that can be as fun, as fulfilling, and as beautiful as art, drama, or music. They may not decide to become mathematicians, but they know that they have the ability to think mathematically and they know what that feels like. And this, after all, was what they wanted in the first place.

Finding Mentors Who Love Math

Eva Owens of Massachusetts writes:

Since I left school last year (what would have been my sophomore year in high school), I have explored several different approaches to

studying math. At first I decided to do math on my own with the help of a Saxon textbook. I discovered that I hadn't gotten a lot out of the freshman algebra class I had taken, so I needed to review a lot of the material. For a while I worked steadily in the textbook and whenever I had a question, I would just ask my father. But I did less and less math work and I eventually stopped in the spring. I lacked enthusiasm for math; it just seemed like an endless stream of meaningless problems with meaningless answers that told me nothing about what mathematics was all about or why I should be studying it (a commonly asked question among students). It was bothering me, though, because I didn't want to lack math and science skills as homeschoolers are sometimes accused of doing; I wanted to be on par with my peers.

During the summer I thought I would get a tutor. I called Susannah Sheffer at GWS for advice as to how to find one. She told me that if I wanted to work on traditional math, the kind of math students do in school, it wouldn't be too hard to find a tutor. I could look on bulletin boards at colleges and libraries, which was what I did. I called up some people who had posted flyers advertising their services, but these people seemed very unenthusiastic about math. This was understandable seeing as they had only taught people who were not looking for enthusiasm; they were looking for a good grade.

Susannah had also mentioned that traditional, school math wasn't the only kind of math I could do. She told me about some articles in back issues of GWS that talked about more unusual math activities that homeschoolers were doing. I thought about this, but at the time, my priority was still simply finding someone who could help me keep up with what my peers were doing in school.

When September came around, I talked with Leslie, a professor who helps me figure out my academic priorities and schedule and who is a general mentor to me. She suggested I post a sign at M.I.T. in Cambridge. I did, and it read: "I'm a 16-year-old homeschooler/independent student living in the Boston area. I'm trying to find a professor of mathematics or a graduate student – with experience working with students – who would be interested in helping me study math. I have a considerable aptitude for mathematics. I need some more direction and structure to help me study further and to help fill in certain gaps in my studies of math. I also want to have the experience of working with someone who has a special enthusiasm and expertise in this field...".

I got calls from two graduate students and set up meetings with both of them. First I met with Alex Coventry who is majoring in number theory. I talked about what my goals were in studying mathematics, which, by then, had become (a) to keep up with my peers with respect to traditional math and (b) to explore non-traditional mathematics and find out what was so great about this subject. Alex was much more inclined to help me with traditional mathematics; his rationale was that I couldn't study any part of mathematics seriously without having first learned the basics. During that meeting he demonstrated his tutoring skills by actually doing a bit of tutoring with me. I liked him a lot and told him I would call soon.

As I walked across the M.I.T. campus to my appointment with Julie Rehmeyer, the other graduate student, I was wondering how I would be able to decide between them if I liked Julie, too. On the phone Julie had told me that she did a little bit of independent studying before she went to college at age 13 and she was very familiar with homeschooling. When we met, she said that she did not want to have anything to do with what is traditionally high school math. She wanted to do something fun and interesting that would help me figure out what mathematics is and see if I liked it. I was so excited because I had stumbled on the perfect combination of help. I didn't decide between the two: I chose them both. Between the two I could accomplish my goals beautifully.

That was October, and now it's February. Recently I wrote two papers, one about why Euclid was so cool and the other about different theories concerning the nature of mathematics. In the second one, I proposed my original idea as to what mathematics is. Several times during these months I have entertained the idea of one day going to M.I.T. or at least seriously pursuing math at the college level. Through studying Euclid and Lobachevsky and just by being around Alex and Julie, two people who are completely in love with math, I have discovered a deep interest in mathematics.

What A Mathematician Does

Julie Rehmeyer of Massachusetts writes:

I am a mathematician, a graduate student in mathematics at the Massachusetts Institute of Technology. I am a strong advocate of homeschooling, and was to some extent a homeschooler myself. One of the reasons I am so excited about homeschooling is to have the chance to teach my children math. So I am always sad to read in GWS about how much many homeschoolers struggle with math.

It's not really very surprising to me, though. Math is incredibly poorly taught in school, often by teachers who dislike the subject or have a poor understanding of it themselves. Furthermore, "school math" has very little relation to how I spend my time as a mathematician. As a result, the beauty of mathematics is often completely inaccessible to those who are learning it. Frankly, I think most people emerge from twelve years of schooling with very little sense of what mathematics is all about.

This is what I think math is all about, and why I love it: first of all, mathematics makes sense. Unfortunately, most of mathematics is taught as a little cookbook: after performing a series of steps, you will get a number which is declared the answer, and if you have done all the steps exactly as told, it is the "right" answer. Students are left mystified about why a particular series of steps is the right one or why the number they get at the end is the right answer. But to me, math is all about understanding. There is an amazing rush when you can say, "Aha! I get it! That's why it works that way!"

Math is beautiful. The beauty is similar to the beauty in music. It

comes from the patterns, the rhythms, the way things fit together just so. The extent of the structure in mathematics is absolutely remarkable. You figure something out to solve one problem, and then find that the technique is just the right one for an entirely different sort of problem. Or you realize that the problem you've been working on is really just one example of a much larger and richer class of things.

Another reason I love math is that it is hard. That may sound perverse, but its sheer difficulty is part of the beauty. There is so much richness in math that no matter how clever you are, you have only perceived the tiniest bit of what is really going on. And the more you know, the more you realize how much you don't know.

What does a mathematician do, anyway? Essentially, we try to understand things about math that no one has ever understood before. The only way to get started on that is to discover for yourself things that other people have already figured out. So if, as you learn math, you are trying to really understand it, to discover it for yourself and make it your own, you are doing exactly what professional mathematicians do.

Another thing many people don't realize is that math is a social sport. Though some mathematicians work shut off in their attics, most don't. I work by talking to people, explaining what I know, asking people questions, trying to understand how someone else thinks about something.

I have been working with a homeschooling student in math recently, and we have had a whole lot of fun. We worked through the first book of *Euclid's Elements,* which is the first place that plane geometry was carefully described. We are just now beginning to study non-Euclidian geometry, which is a remarkable theory of a completely different kind of geometry. The development of non-Euclidian geometry changed how even mathematicians thought about what mathematics fundamentally is. So the student and I are following that development and thinking those questions through for ourselves.

Here are a few books I recommend: *A Mathematician's Apology* by G.H. Hardy gives a far better description of what math is like than I have done here. It is entirely non-mathematical in content, but simply tries to explain what math is about and why society should have pure mathematicians at all. *Gödel, Escher, Bach* by Douglas Hofstadter is huge and interesting. It can be somewhat demanding reading, but it requires little previous knowledge and has a wonderful annotated bibliography. *The Emperor's New Mind* by Roger Penrose is also very good. It is primarily a discussion of the extent to which computers can model the human brain, but it wanders into many related topics. *Gödel's Proof* by Nagel and Newman is an accessible exposition of a remarkable theorem in logic which has broad philosophical ramifications. Finally, Morris Kline has written a whole slew of books on the history and culture of mathematics. The ones I happen to have are *Mathematical Thought from Ancient to Modern Times,* which explains the evolution of mathematical ideas, and *Mathematics in Western Culture,* about the influence of mathematics on other areas of thought.

Unfortunately, none of these books is accessible to young children, but their parents might find them interesting for the broader view of mathematics they present.

Unschooling Math

Homeschoolers and their parents share their challenges and solutions to learning math using a variety of materials, and interactions - but rarely math textbooks!

This useful supplement helps you learn math in more interesting, meaningful, and practical ways.

These stories are selected from Growing Without Schooling magazine's archives. GWS is the nation's oldest homeschooling publication, founded in 1977 by the late author and teacher, John Holt.

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